R&D Cycle in a Fully-endogenous Growth Model with Population Growth

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Abstract

This paper presents that there exists a fully-endogenous R&D cycle in an R&D-growth model with exogenous labor growth and without knowledge externalities. Assuming two types of R&D: horizontal R&D (uses capital) and vertical R&D (uses labor), there is a period 2 cycle in which the economy faces with relatively high R&D expenditures and with relatively low R&D expenditures reciprocally. The source of this cyclical behavior is the market structures in intermediates sector. The long-run average per-capita growth rate in the cycle is not determined by the labor growth rate, but by vertical R&D intensity measured by the ratio of engineers over labor. In the period 2 cycle, the economy also faces with stable average growth rate beyond ages and stationary upward trend in R&D expenditures. In addition to the above characteristics, there exists an R&D promotion policy which can increase both long-run and short-run growth rate.

Keywords: R&D, Growth cycle, Endogenous growth
JEL codes: O40, O31, O41

1 Introduction

Innovation is considered to play a central role in long-run economic growth through the improvement of productivity. Most advanced countries struggle in promoting innovation activity for increasing the growth rate. In innovation activities, many fields, including marketing, management, and history of technology, observe various types of cyclical behavior. In marketing literature, product life cycle is widely observed. Concerning economics, from Schumpeter's "creative destruction" to medium-term cycle cyclical behavior is closely related to firm's innovation activity (i.e. R&D).

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This paper constructs an empirically plausible growth cycle model that contains upward trend in R&D expenditure and endogenously determined stable average growth rate beyond ages without complex and stochastic assumptions in R&D sector. To accomplish these plausibilities, we assume two types of endogenously determined R&D: horizontal R&D which is the source of cyclical behavior and vertical R&D which is the main drive force of long-run economic growth. In order to make the model analyzable, we assume that horizontal R&D promotes capital productivity and uses capital, and vertical R&D promotes labor productivity and uses labor (engineer).

From a theoretical viewpoint, the growth cycle theory analyzes the endogenous cyclical behavior in R&D and economic growth. Many previous studies in growth cycle theory consider counter-cyclical R&D cycle, where when the economy faces high R&D expenditures, the growth rate of the economy is lower. However, empirical works in the relationship between R&D expenditures and growth from the viewpoint of growth cycle such as Walde and Woitek (2004) pointed out that in the real economy the typical relation of such two variables is pro-cyclical (i.e. when R&D expenditures of the economy are high the growth rate is also high). Some recent works such as Walde (2005) and Barlevy (2007) assume a stochastic process in R&D and explain pro-cyclical R&D cycle. Moreover, Comin and Gertler (2006) introduce diffusion lags in technology adoption in the DSGE model to explain the pro-cyclical R&D cycle. Shinagawa (2013) assume a high rate of population growth and a negative externality in R&D so as to make counter-cyclical R&D expenditures pro-cyclical. Yano and Kanehara (2016) and Kanehara (2016) are the studies in the growth cycle theory, which consider the effect of labor growth to the R&D and growth rate. In their work, depending on the parameter conditions there exist both pro-cyclical R&D cycle and counter-cyclical R&D cycle without assuming the stochastic process and negative externality.

Unfortunately, their model is a semi-endogenous growth model[1] in which the growth rate in the balanced growth path and the average growth rate in the cycle are not determined endogenously but by exogenously growing input. However, empirical such as like Ha and Howitt (2007) and Madsen (2008) pointed out that semi-endogenous growth models are not consistent with the recent developed countries data. They also evaluate the first-generation R&D growth models[2] and fully-endogenous Schumpeterian models[3], and conclude that fully-endogenous Schumpeterian models are more consistent with the data than the other two types. Another problem lies in the stational R&D expenditures both in the balanced growth path and in period 2 cycle. On the other hand, Jones (1995a) pointed out that despite the stationality of the growth rate, R&D expenditures

[1]Jones (1995), Kortum (1997), and Segerstrom (1998) are the seminal papers of the semi-endogenous growth model.

[2]Romer (1990), Aghion and Howitt (1992), and Grossman and Helpman (1993) are the seminal papers of this first generation. Although they require scale effect, their steady growth rates are determined endogenously.

[3]Dinopoulos and Thompson (1998), Peretto (1998) and Howitt (1999) are the seminal papers of this series of studies. They require positive population growth rate, and do not assume the scale effect. Their steady growth rates are determined endogenouly.

in advanced countries after the WW2 has upward trends.

Although the empirical plausibility of fully-endogenous Schumpeterian models, there exists a severe critique in their assumption. This critique is often called "knife-edge critique" pointed out Jones (1999). He claimed that to remove the scale effect from the steady equilibrium, they assume specific production function in horizontal R&D so as to offset the scale effect in vertical R&D. On the other hand, if they employ more general function form, fully-endogenous Schumpeterian models still exhibit scale effects. To extend this knife-edge critique Jones (2005) criticize the robustness of most of all endogenous growth model for assuming linear production structure in endogenously growing state variables because the result of such models can be fragile if production is less than linear. To avoid the latter "linearity critique" it will be difficult, except a few recent studies like Peretto (2016), to treat endogenous growth when the key difference equation of the models is less than linear. On the other hand, avoiding the former knife-edge critique would be possible if we assume that each type of R&D uses different inputs.

Based on the above discussion, we introduce a vertical R&D into Yano and Kanehara (2016) and construct a fully-endogenous growth cycle model which contains stable average growth rate and stationary growing R&D expenditures, eliminates scale effect in each R&D sectors and avoids the knife-edge critique. The key to making this model analyzable lies on the assumption of the inputs of each R&D. We assume that in order to make each R&D activity independent each R&D uses different R&D input. The rest of this paper is organized as follows: in Section 2, we set up the model and discuss the three factors to promote growth: horizontal R&D, population growth, and vertical R&D. In Section 4, we analyze the dynamical system, characterize the balanced growth path and the period 2 cycle, and examine the effects of both long-run and short-run R&D promotion policy. Finally, we compare the relationship between the local stability of the balanced growth path, the existence of the period-2 cycle, and the condition of pro-cyclical R&D cycle.

2 The Model

We present the model, introducing the vertical R&D to Yano and Kanehara (2016). Since Yano and Kanehara (2016) is a Matsuyama (1999)-type growth cycle model with exogenously growing efficient labor, we also employ the same assumption as Matsuyama (1999) in the horizontal R&D. The assumption about the vertical R&D is similar to the “fully-endogenous Schumpeterian endogenous growth model” like Dinopoulos and Thompson (1998), Peretto (1998) and Howitt (1999), although we do not assume the “creative destruction” structures in vertical R&D.

Concerning the household, following Matsuyama (1999), we consider fixed saving rate μ in this model. As Matsuyama (1999) mentioned, this assumption is justified because of an optimal saving decision by two-period-lived overlapping generations consumers. In addition to this, we assume the income tax τ as it follows:

$$K_{t+1} = \mu(1-\tau)Y_t, \tag{1}$$

where K_{t+1} denote the aggregate capital stock available for production and innovation in period t and Y_t is the quantity of final good (consumable capital good).

2.1 Production

Although the final good production uses intermediate goods and labor, the level of efficiency in this sector determined by the vertical R&D. The vertical R&D is achieved by the final good producer using labor. The final good is consumable capital good traded in a competitive[4] market, which is the input for the horizontal R&D and intermediate goods production and treated as a numeraire. The intermediate goods sector includes horizontal R&D firms and intermediate goods producers, which are faced with different market structures[5].

The production function of the final good in period t is

$$Y_t = (A_t l_t)^\gamma \left(\int_0^{N_t} (x_t(i))^\alpha di \right)^\beta, \alpha = 1 - \frac{1}{\varepsilon}, \varepsilon \in (1,\infty),\ \beta, \gamma \in (0,1), \beta + \gamma = 1, \tag{2}$$

where A_t is a Harrod neutral technology level in the final good sector, l_t is the quantity of workers using in production, and $x_t(i)$ denotes the quantity of the ith type of the intermediate good in period t. $[0, N_t]$ denote the varieties of the intermediate goods available in this period. We denote $p_t(i)$ as the price of the ith intermediate good at period t. To simplify the discussion, we denote $X_t = \int_0^{N_t} (x_t(i))^\alpha di$ as the index of $x_t(i)$ and $P_t := \left[\int_0^{N_t} [p_t(i)]^{\frac{\alpha}{\alpha-1}} di \right]^{\frac{\alpha-1}{\alpha}}$ as the price index of the intermediates.

Similar to Matsuyama (1999) in each period t, intermediates $i \in [0, N_{t-1}]$ are traded in competitive market, and on the other hand, intermediates $i \in [N_{t-1}, N_t]$ are invented and sold exclusively by their innovators. By symmetry, we denote $x_t(i) := x_t^c, p_t(i) := p_t^c$ for competitive intermediates and $x_t(i) := x_t^m, p_t(i) := p_t^m$ for monopolistic intermediates. In addition, we introduce a sort of bounded rationality[6] used in Yano and Kanehara (2016). By the optimization

[4] It seems unusual to assume R&D in a competitive market. This, however, can be justified by the existence of potential competitors outside of the market. More concretely, only the final good producer with the highest technology level can produce the final good. In order to avoid entry of potential competitors, the final good producer act as if she is in a competitive market.

[5] The market of horizontal R&D firm is monopolistic, and that of intermediate goods producers is competitive.

[6] First final good producer decides $x_t(z)$ so as to minimize the cost for given the index of the quantities of the intermediate goods $X_t = \int_0^{N_t} (x_t(i))^\alpha di$. To do so, $x_t(i)$ is determined by X_t, and $p_t(i)$ and the price index $P_t = \left[\int_0^{N_t} [p_t(i)]^{\frac{\alpha}{\alpha-1}} di \right]^{\frac{\alpha-1}{\alpha}}$. By $x_t(i)$ we can derive X_t. After that, the producer's profit maximization problem faces with the Cobb-Douglas production function about X_t and l_t.

of this sector, Each $x_t(i)$ satisfies

$$\frac{x_t^c}{x_t^m} = \left(\frac{p_t^c}{p_t^m}\right)^{\frac{1}{\alpha-1}}. \tag{3}$$

In addition to Yano and Kanehara (2016), our final good sector can improve its technology level through vertical R&D. The input of vertical R&D is efficient labor, and the decision of the degree of this activity is achieved after the solving the profit maximization problem w.r.t. $x_t(i)$ and l_t.

Denoting w_t as the wage of labor, based on above discussion, the maximization problem of final good production can be rewritten as follows

$$\max_{X_t, l_t} A_t^\gamma X_t^\beta l_t^\gamma - P_t X_t^{\frac{1}{\alpha}} - w_t l_t.$$

By the FOCs, we get

$$w_t = \gamma \frac{Y_t}{l_t}, P_t = \alpha\beta \frac{Y_t}{X_t^{\frac{1}{\alpha}}}. \tag{4}$$

In this case, the maximized profit of the producer Π_t is $(1-\alpha\beta-\gamma)Y_t$. By the assumption of α, β, γ, the producer has strictly positive profit. We assume that the government collects all of the profit via corporation tax after the above optimization. Tax $(1-\alpha\beta-\gamma)Y_t$ are ex-post redistribute to the final good producer as a subsidy for vertical R&D. Moreover, we assume another R&D subsidy ς from income taxiation τY_t so as to promote vertical R&D. Subsidy ς is determined by R&D policy via the controlling the rate of income taxiation τ. Therefore, the total amount of vertical R&D becomes $(1-\alpha\beta-\gamma+\tau)Y_t$. In the usual fully-endogenous Schumpeterian model, the incentive for vertical R&D is "creative destruction" that is, R&D race and Bertrand competition in the vertical innovators. However, since such an assumption would make the analysis more complicated, in this paper, we do not adopt the "creative destruction" assumption and simply assume that vertical R&D is operated by the final good producer and the incentive is to avoid taxation[7].

2.2 Horizontal R&D

Both innovators and intermediate producers produce one unit of intermediate by using a units of capital[8]. To innovating "new" intermediates, each innovator requires F units of capital and enjoys temporal (one period) monopolistic power about their innovation. We denote r_t as the price of capital in period t, and thus, the marginal cost of producing intermediate goods is ar_t. Considering the market structures of producers and innovators, prices of intermediate goods p_t^c and p_t^m satisfy the following:

$$p_t^c = ar_t, p_t^m = \frac{ar_t}{\alpha}. \tag{5}$$

[7] Although our model is not a Schumpeterian model, the balanced growth path and the period-2 cycle are still fully-endogenous and free from the scale effect.

[8] Their production function faces constant returns to scale.

The incentive of innovation is to obtain one-period monopoly power about its product and we assume that there is no barrier to entry for innovation. Since the monopoly profit of the innovators is $\pi_t^m = p_t^m x_t^m - a x_t^m r_t - F$ and the profit of the intermediate producers is zero, innovation occurs in period t[9] if

$$x_t^m \leq \frac{\alpha}{a(1-\alpha)} F$$

In equilibrium, by the free-entry assumption, the monopoly profit is equal to that of producers if the economy has innovation in period t, we have the following condition

$$x_t^m = \frac{\alpha}{a(1-\alpha)} F. \tag{6}$$

The capital constraint in period t can be written as

$$K_{t-1} = \frac{1}{1-\alpha} \left(N_{t-1} x_t^c + \left(N_t - N_{t-1} \right) x_t^m \right) F.$$

By the above discussion, this can be rewritten as

$$\begin{aligned} K_{t-1} &= \frac{1}{1-\alpha} \left[N_{t-1} \alpha^{\frac{\alpha}{\alpha-1}} + \left(N_t - N_{t-1} \right) \right] x_t^m F \\ &= N_{t-1} + \max \left\{ 0, \frac{1-\alpha}{F} K_{t-1} - \alpha^{\frac{\alpha}{\alpha-1}} N_{t-1} \right\}. \end{aligned}$$

By the capital constraint, the degree of variety expansion (horizontal R&D) is

$$N_t = N_{t-1} + \max \left\{ 0, \frac{1-\alpha}{F} K_{t-1} - \alpha^{\frac{\alpha}{\alpha-1}} N_{t-1} \right\} \tag{7}$$

2.3 Population Growth and Vertical R&D

In this model, in addition to Matsuyama (1999), we assume exogenous population (efficient labor) growth and vertical R&D, the input of which is only labor. They are other driving forces of sustainable growth.

First, we assume that the economy faces with exogenous population (=labor) L_t growth

$$L_t = m L_{t-1}, m > 1 \text{ for } \forall t. \tag{8}$$

Different from previous Matsuyama (1999)-type growth cycle model, this L_t is not inelastically supplied to the final good production. We assume that although labor L_t is fully utilized, some of them become workers l_t for the final good, and others become engineers l_t' $(= L_t - l_t)$ and work for vertical R&D. To simplify the notation, we denote the relative rate of engineers over workers $h_t := \frac{l_t'}{l_t}$. This is a measure of vertical R&D intensity. Vertical R&D is an activity to promote the Harrod-neutral technology level A_t in the final good sector. The evolution of A_t is characterized by the following equation

$$A_t = \lambda^{\frac{l_t'}{l_t}} A_{t-1} = \lambda^{h_t} A_{t-1}, \ \lambda > 1. \tag{9}$$

[9] That is, $N_t > N_{t-1}$

Comparing the technology level in period t A_t and initial technology level A_t, we can derive

$$A_t = \lambda^{\sum_{t=0}^{t} h_t}.$$

The efficiency of vertical R&D decreases at the same rate as the number of workers increases. The intuition of this assumption is that since an economy with larger number of workers has higher diversity, as the number of workers of the economy increases it is more difficult for engineers to improve manufacturing technology which is appliable to all workers.

Since the technology level A_t is determined by h_t, we need to derive h_t from the job selection. Assuming that the cost of labor is indifferent in each occupation, only the wage can decide the relative ratio h_t. Since the wage of workers in period t is $w_t = \gamma \frac{Y_t}{l_t}$ and fiscal resources for employing engineers are $(1-\alpha\beta-\gamma+\tau)Y_t$, the equilibrium rate of h_t is determined by income equalization (i.e. both wages of worker and engineers become w_t in equilibrium). Therefore, the number of engineers who work for vertical R&D is

$$l'_t = \frac{(1-\alpha\beta-\gamma+\tau)Y_t}{w_t} = \frac{(1-\alpha\beta-\gamma+\tau)w_t l_t}{\gamma w_t}. \tag{10}$$

The vertical R&D intensity $h_t = \frac{l'_t}{l_t}$ becomes

$$h_t = \frac{1-\alpha\beta-\gamma+\tau}{\gamma}. \tag{11}$$

There is no time-dependent factor in the rate h_t, and thus, we rewrite h_t as $h := \frac{1-\alpha\beta-\gamma+\tau}{\gamma}$. If there are no policies which can change the income tax τ over time, the technology level A_t is described as

$$\begin{aligned} A_t &= \lambda^h A_{t-1} \\ &= \lambda^{ht} \end{aligned}$$

Since

$$\frac{\partial h_t}{\partial \tau} = \frac{1}{\gamma} > 0,$$

the increase in τ increases h_t and l'_t, and, therefore, also increases A_t. This implies that R&D promotion policy can stimulate vertical R&D. To simplify the discussion we assume the temporal effect of income taxiation policy to the output:

$$\frac{\partial Y_t}{\partial \tau} < 0, \frac{\partial Y_{t+1}}{\partial \tau} + \frac{\partial Y_{t+1}}{\partial A_t}\frac{\partial A_t}{\partial \tau} + \frac{\partial Y_t}{\partial \tau} > 0. \tag{12}$$

These assumption imply that the increase of the ratio h_t in period t stimulates vertical R&D, however, since the increase of income tax τ obstruct capital accumulation, R&D promotion policy does not directly stimulate growth rate in the same period.

To summarize the result in production, we have the following final good production function

$$Y_t = \begin{cases} \left(\frac{1-\alpha}{F}\left(\frac{\alpha F}{a(1-\alpha)}\right)^{\alpha}\right)^{\beta} K_{t-1}^{\beta}\left(\frac{1}{1+h}A_tL_t\right)^{\gamma} & \text{if} \quad K_{t-1} \geq \left(\frac{\alpha^{\frac{\alpha}{\alpha-1}}}{1-\alpha}\right) FN_{t-1} \\ \left(\frac{1}{a}\right)^{\alpha\beta} N_{t-1}^{\beta(1-\alpha)} K_{t-1}^{\alpha\beta}\left(\frac{1}{1+h}A_tL_t\right)^{\gamma} & \text{if} \quad K_{t-1} \leq \left(\frac{\alpha^{\frac{\alpha}{\alpha-1}}}{1-\alpha}\right) FN_{t-1} \end{cases} \tag{13}$$

3 The Dynamical System

3.1 State Variables and Normalization

The state variables in the model are capital K_t, variety N_t, labor L_t, and technology level A_t. We need to examine the dynamics of each state variable and consider how to derive the steady state (balanced growth path). By (7)(8)(9)(12), we have the following relations

$$\begin{cases} K_t = \begin{cases} \mu(1-\tau)\left(\frac{1-\alpha}{F}\left(\frac{\alpha F}{a(1-\alpha)}\right)^{\alpha}\right)^{\beta} K_{t-1}^{\beta}\left(\frac{1}{1+h}A_tL_t\right)^{\gamma} & \text{if} \quad K_{t-1} \geq \left(\frac{\alpha^{\frac{\alpha}{\alpha-1}}}{1-\alpha}\right) FN_{t-1} \\ \mu(1-\tau)\left(\frac{1}{a}\right)^{\alpha\beta} N_{t-1}^{\beta(1-\alpha)} K_{t-1}^{\alpha\beta}\left(\frac{1}{1+h}A_tL_t\right)^{\gamma} & \text{if} \quad K_{t-1} \leq \left(\frac{\alpha^{\frac{\alpha}{\alpha-1}}}{1-\alpha}\right) FN_{t-1} \end{cases}, \\ N_t = N_{t-1} + \max\left\{0, \frac{1-\alpha}{F}K_{t-1} - \alpha^{\frac{\alpha}{\alpha-1}}N_{t-1}\right\}, \\ L_t = mL_{t-1}, \\ A_t = \lambda^{h_t}A_{t-1}. \end{cases} \tag{14}$$

The R&D expenditure of the economy including both types of R&D is $E_t = (1-\alpha\beta-\gamma+\tau)Y_t + (N_t - N_{t-1})F$. Considering the pro-cyclicality of R&D expenditure, this index E needs to increase when the growth rate becomes higher, and vice versa. When the period faces positive horizontal R&D that is, $K_{t-1} \geq \left(\frac{\alpha^{\frac{\alpha}{\alpha-1}}}{1-\alpha}\right) FN_{t-1}$, we say that the economy is in the Romer regime. On the other hand, when the period faces with no horizontal R&D i.e. $K_{t-1} < \left(\frac{\alpha^{\frac{\alpha}{\alpha-1}}}{1-\alpha}\right) FN_{t-1}$, we say that the economy is in the Romer regime.

When we analyze the dynamical system, we easily recognize that L_t and A_t are growing exogenously, and thus, it is necessary to normalize other variables K_t, N_t by those two exogenously growing L_t and A_t. The most simple and intuitive way is to redefine new variables about capital and variety, which contain both L_t and A_t. Therefore, we denote a new state variable $\Lambda_t := L_tA_t$ and normalize K_t, N_t by Λ_t. Denoting $k_t := \frac{K_t}{\Lambda_t}, n_t := \frac{N_t}{\Lambda_t}$, the dynamical system of the model can be described by the following the difference equation $(k_t, n_t) =$

$\Phi(k_{t-1}, n_{t-1})$:

$$\begin{bmatrix} k_t \\ n_t \end{bmatrix} = \begin{cases} \begin{bmatrix} \mu(1-\tau)\frac{1}{(1+h)^\gamma}\left(\frac{1-\alpha}{F}\left(\frac{\alpha F}{a(1-\alpha)}\right)^\alpha\right)^\beta \frac{k_{t-1}^\beta}{m\lambda^h} \\ \left(1-\alpha^{\frac{\alpha}{\alpha-1}}\right)\frac{n_{t-1}}{m\lambda^h} + \frac{1-\alpha}{F}\frac{k_{t-1}}{m\lambda^h} \end{bmatrix}, & \text{if} \quad k_{t-1} \geq \left(\frac{\alpha^{\frac{\alpha}{\alpha-1}}}{1-\alpha}\right) F n_{t-1} \\ \begin{bmatrix} \mu(1-\tau)\frac{1}{(1+h)^\gamma}\frac{1}{a^{\alpha\beta}}\frac{n_{t-1}^{\beta(1-\alpha)}k_{t-1}^{\alpha\beta}}{m\lambda^h} \\ \frac{n_{t-1}}{m\lambda^h} \end{bmatrix}, & \text{if} \quad k_{t-1} \leq \left(\frac{\alpha^{\frac{\alpha}{\alpha-1}}}{1-\alpha}\right) F n_{t-1}, \end{cases} \quad t = 1, 2, 3\cdots. \tag{15}$$

3.2 Balanced Growth Path

In this section, we derive the steady state and the period-2 cycle in this difference equation. In order to simplify the notation, we denote $B = \left(\frac{\alpha^{\frac{\alpha}{\alpha-1}}}{1-\alpha}\right) F, C = \mu(1-\tau)\frac{1}{(1+h)^\gamma}\left(\frac{1-\alpha}{F}\left(\frac{\alpha F}{a(1-\alpha)}\right)^\alpha\right)^\beta$. Therefore, we can rewrite (14) as

$$\begin{bmatrix} k_t \\ n_t \end{bmatrix} = \begin{cases} \begin{bmatrix} \frac{Ck_{t-1}^\beta}{m\lambda^h} \\ \frac{n_{t-1}}{m\lambda^h}\left(1+\alpha^{\frac{\alpha}{\alpha-1}}\left(\frac{k_{t-1}}{Bn_{t-1}}-1\right)\right) \end{bmatrix}, & \text{if} \quad k_{t-1} \geq Bn_{t-1} \\ \begin{bmatrix} CB^{\beta(1-\alpha)}\frac{n_{t-1}^{\beta(1-\alpha)}k_{t-1}^{\alpha\beta}}{m\lambda^h} \\ \frac{n_{t-1}}{m\lambda^h} \end{bmatrix}, & \text{if} \quad k_{t-1} \leq Bn_{t-1}, \end{cases} \quad t = 1, 2, 3\cdots. \tag{16}$$

The dynamical system has a unique steady state (balanced growth path) (k^*, n^*), where $k^* \geq Bn^*$ is satisfied and

$$k^* = \left(\frac{C}{m\lambda^h}\right)^{\frac{1}{1-\beta}}, n^* = \frac{\left(\frac{C}{m\lambda^h}\right)^{\frac{1}{1-\beta}}}{\left[\frac{1}{\alpha^{\frac{\alpha}{\alpha-1}}}\left(m\lambda^h - 1\right) + 1\right]B}.$$

In this balanced growth path (BGP), growth rates of GDP g_Y, capital g_K, variety g_N, labor g_L, and technology level g_A are

$$g_Y = g_K = g_N = m\lambda^h, g_L = \frac{L_t}{L_{t-1}} = m, \text{ and } g_A = \frac{A_t}{A_{t-1}} = \lambda^h.$$

Per-capita GDP growth rate g_y in the BGP becomes

$$g_y = \frac{g_Y}{g_L} = \lambda^h.$$

Therefore, BGP growth rate is determined by labor growth rate m and growth rate of technology level λ^h. On the other hand, per-capita GDP growth rate is determined by growth rate of technology level λ^h only. Since technology level λ^h is endogenously determined, the growth rate in BGP is fully endogenous. We denote R&D expenditure in period t E_t as

$$E_t = (1 - \alpha\beta - \gamma + \tau)Y_t + (N_t - N_{t-1})F. \tag{17}$$

The growth rate of R&D expenditure g_e in the BGP becomes

$$\begin{aligned} g_e &= \frac{E_t}{E_{t-1}} = \frac{(1-\alpha\beta-\gamma+\tau)Y_t + (N_t - N_{t-1})F}{(1-\alpha\beta-\gamma+\tau)Y_{t-1} + (N_{t-1} - N_{t-2})F} \\ &= m\lambda^h. \end{aligned}$$

Different from Yano and Kanehara (2016) $\frac{E_t}{E_{t-1}}$ has stationary upward trend in proportion labor growth rate and this is consistent with Jones (1995a).

To analyze the local stability of the BGP, the Jacobian of the difference equation $\Phi(k_{t-1}, n_{t-1})$ at (k^*, n^*) is

$$\begin{aligned} J &= \begin{bmatrix} \frac{\partial \Phi_1^R}{\partial k^*}(k^*, n^*) & \frac{\partial \Phi_1^R}{\partial n^*}(k^*, n^*) \\ \frac{\partial \Phi_2^R}{\partial k^*}(k^*, n^*) & \frac{\partial \Phi_2^R}{\partial n^*}(k^*, n^*) \end{bmatrix} \\ &= \begin{bmatrix} \beta & 0 \\ \frac{\alpha^{\frac{\alpha}{\alpha-1}}}{Bm\lambda^h} & \frac{1-\alpha^{\frac{\alpha}{\alpha-1}}}{m\lambda^h} \end{bmatrix} \end{aligned}$$

Therefore, the eigenvalues are β and $\frac{1-\alpha^{\frac{\alpha}{\alpha-1}}}{m\lambda^{1-h}}$, and since $\beta \in (0,1)$, we focus on the condition of $\left|\frac{1-\alpha^{\frac{\alpha}{\alpha-1}}}{m\lambda^h}\right| < 1$. Using $\alpha = 1 - \frac{1}{\varepsilon}, \varepsilon \in (1, \infty)$, this can be rewritten as

$$\left(1 - \frac{1}{\varepsilon}\right)^{1-\varepsilon} - 1 < m\lambda^h.$$

Since this condition is not clear, we consider the limit case $\varepsilon \to \infty$,

$$e - 1 < m\lambda^h. \tag{18}$$

Therefore, the local stability of the BGP requires a high rate of both labor growth rate m and vertical R&D growth rate.

3.3 The Effects in Changing Parameters

First, we examine the temporal (and marginal) effect of changing parameters (and pre-determined variables) to capital K_t, variety N_t, labor L_t, and technology level A_t. To consider such changes in parameters, we can examine the policy implication of the model. We can summarize such effects in the following table 1:

Romer regime	K_t	N_t	L_t	A_t	Solow regime	K_t	N_t	L_t	A_t
F	+	-	0	0	F	0	0	0	0
τ	-	0	0	+	τ	-	0	0	+
K_{t-1}	+	+	0	0	K_{t-1}	+	0	0	0
N_{t-1}	0	-	0	0	N_{t-1}	+	1	0	0
L_t	+	0	1	0	L_t	+	0	1	0
A_{t-1}	+	0	0	+	A_{t-1}	+	0	0	+

table 1

Especially we focus on the following two R&D promotion policies[10]: deregulation in horizontal R&D sector which decreases horizontal R&D cost F and vertical R&D subsidies τ which support engineers's wages. Based on the above table, we can understand that both the decrease in F (when the economy is in the Romer regime) and the increase in τ promote horizontal and vertical R&D in the short-run respectively. The effects to K_t are also identical. Deregulation in period t obstructs capital accumulation (therefore decrease GDP growth rate) through the increase in N_t at that period, however the increase in N_t promotes capital accumulation in period $t+1$ (and does not obstruct horizontal R&D in period $t+1$). Similary since vertical R&D is not productive, the temporal increase in τ obstruct capital accumulation in this period, however, the increased A_t promotes the capital accumulation in period $t+1$ (and does not obstruct vertical R&D in period $t+1$). Therefore, each temporal R&D promotion policies initially promote R&D and obstruct capital accumulation. Then they promote capital accumulation in the next period. By assumption the effect of R&D subsidies in period t increases GDP growth rate from period $t+1$ on.

Next, we consider the long-run effect of permanent R&D promotion policies around the BGP[11]. Different from short-run (and temporal), R&D promotion policies have heterogenous effect. Deregulation has no effect in long-run growth rate in the BGP. On the other hand, since vertical R&D subsidies increase h, subsidies also increase the equilibrium growth rate $m\lambda^h$. Interestingly, since $g_N = m\lambda^h$, vertical R&D subsidies also promote horizontal R&D in the long-run. Moreover, considering that growth rate of R&D expenditure in the balanced growth path satisfies $\frac{E_t}{E_{t-1}} = m\lambda^h$, R&D subsidies also stimulate R&D expenditure.

Considering these policy implications and the endogenously determined growth rate in BGP, our model is not a semi-endogenous growth model but a fully endogenous growth model like Dinopoulos and Thompson (1998), Peretto (1998) and Howitt (1999). Different from fully endogenous growth models our model is free from the assumption of knowledge externality, therefore, our model does not faces with the case where R&D promotion policy can retard long-run economic growth pointed out in Segerstrom (2000).

3.4 Period 2 Cycle

Similar to the Matsuyama (1999)-type growth cycle model, because of the structure of intermediate goods sector and horizontal R&D, this model also faces nonmonotonicity in the dynamics of N_t. Unfortunately, since we cannot describe the dynamical system as a one-dimensional map, it is not easy to analyze the nonmonotonicity intuitively. However, like Yano and Kanehara (2016), we can derive the period 2 cycle, where (k_t, n_t) fluctuates forever between the Romer

[10] We intentionally do not consider horizontal R&D subsidies, This is partly because our model is a sort of "semi-endogenous growth model", but also Aloi and Lasselle (2007) and Li and Zhang (2014) already analyzed such subsidizing policy in Matsuyma (1999)-type growth cycle model.

[11] The long-run policy implications discussed below can be applied to the period 2 cycle.

and Solow regimes. In the period 2 cycle we assume that (k_t, n_t) alternates between $\left(k^R, n^R\right)$ and $\left(k^S, n^S\right)$. The $\left(k^R, n^R\right)$ and $\left(k^S, n^S\right)$ satisfy

$$
\begin{aligned}
k^S &= \frac{Ck^{R\beta}}{m\lambda^h} CB^{\beta(1-\alpha)} \frac{n^{S\beta(1-\alpha)} k^{R\alpha\beta}}{m\lambda^h}, \\
n^S &= \frac{n^R}{m\lambda^h}\left(1 + \alpha^{\frac{\alpha}{\alpha-1}}\left(\frac{k^R}{Bn^R} - 1\right)\right)\frac{n^S}{m\lambda^h}.
\end{aligned}
$$

One of the examples to satisfy this condition is the following $\left(k^R, n^R\right)$ and $\left(k^S, n^S\right)$

$$
\begin{aligned}
k^R &= \left[\left(\frac{C}{m\lambda^h}\right)^{1+\beta} B^{\beta(1-\alpha)} \left(n^S\right)^{\beta(1-\alpha)}\right]^{\frac{1}{1-\alpha\beta^2}}, \\
n^R &= \frac{1}{m\lambda^h}\left[\frac{\alpha^{\frac{\alpha}{\alpha-1}} C^{\frac{1+\alpha\beta}{1-\alpha\beta^2}} B^{\frac{\beta(1-\alpha)-1+\alpha\beta^2}{1-\alpha\beta^2}}}{\left(m^2\lambda^{2h} - 1 + \theta\right)\left(m\lambda^h\right)^{\frac{\alpha\beta(1+\beta)}{1-\alpha\beta^2}}}\right]^{\frac{1-\alpha\beta^2}{1-\alpha\beta^2-\beta(1-\alpha)}},
\end{aligned}
$$

$$
\begin{aligned}
k^S &= \frac{C}{m\lambda^h}\left[\left(\frac{C}{m\lambda^h}\right)^{1+\beta} B^{\beta(1-\alpha)} \left(n^S\right)^{\beta(1-\alpha)}\right]^{\frac{1}{1-\alpha\beta^2}}, \\
n^S &= \left[\frac{\alpha^{\frac{\alpha}{\alpha-1}} C^{\frac{1+\alpha\beta}{1-\alpha\beta^2}} B^{\frac{\beta(1-\alpha)-1+\alpha\beta^2}{1-\alpha\beta^2}}}{\left(m^2\lambda^{2h} - 1 + \theta\right)\left(m\lambda^h\right)^{\frac{\alpha\beta(1+\beta)}{1-\alpha\beta^2}}}\right]^{\frac{1-\alpha\beta^2}{1-\alpha\beta^2-\beta(1-\alpha)}}.
\end{aligned}
$$

The existence of the period 2 cycle is guaranteed when

$$
2 - \alpha^{\frac{\alpha}{\alpha-1}} \leq m^2\lambda^{2h} \leq 1 + \frac{1}{\mu C} - \alpha^{\frac{\alpha}{\alpha-1}}. \tag{19}
$$

Since $2 - \alpha^{\frac{\alpha}{\alpha-1}}$ is smaller than one, this becomes $m\lambda^h \leq \left(1 + \frac{1}{\mu A} - \alpha^{\frac{\alpha}{\alpha-1}}\right)^{\frac{1}{2}}$. In contrast to the local stability of the BGP, the inequality requires both low labor growth rate m, and low vertical R&D growth rate λ^h.

In the period 2 cycle, the average GDP, capital, and variety growth rate are also $m\lambda^h$. Since the economy fluctuates forever between the Romer and Solow regimes in the period 2 cycle, the growth rates of technology level are $g_A^R = m^2\lambda^{2h}$ and $g_A^S = 0$. The GDP, and capital growth rates in each period are

$$
\begin{aligned}
g_Y^R &= g_K^R = C(m^{t+2}\lambda^{h(t+2)}L_0)^{1-\beta} K_{t+1}^{\beta-1}, \\
g_Y^S &= g_K^S = CB^{\beta(1-\alpha)}(m^{t+1}\lambda^{h(t+2)}L_0)^{\gamma+\beta} N_t^{\beta(1-\alpha)} K_t^{\alpha\beta-1}.
\end{aligned}
$$

Since $g_Y^R g_Y^S = m^2\lambda^{2h}$, g_Y^S and g_K^S are rewritten by

$$g_Y^S = g_K^S = \frac{m^2\lambda^{2h}}{C(m^{t+2}\lambda^{h(t+2)}L_0)^{1-\beta}K_{t+1}^{\beta-1}}.$$

If t is large enough to satisfy $K_{t+1} \simeq m^{t+1}\lambda^{h(t+1)}K_0, K_t \simeq m^t\lambda^{ht}K_0$, g_Y^R and g_Y^S become

$$\begin{aligned} g_Y^R &= C(m\lambda^h L_0)^{1-\beta}K_0^{\beta-1}, \\ g_Y^S &= \frac{m^{\beta+1}\lambda^{h(\beta+1)}}{AL_0^{1-\beta}K_0^{\beta-1}}. \end{aligned}$$

The period 2 cycle becomes a pro-cyclical R&D cycle when $g_Y^R > g_Y^S$. This condition is satisfied if and only if

$$m\lambda^h < \left[\mu\left(\frac{1-\alpha}{F}\left(\frac{\alpha F}{a(1-\alpha)}\right)^{\alpha}\right)\beta L_0^{1-\beta}K_0^{\beta-1}\right]^{\frac{1}{\beta}}. \tag{20}$$

Therefore, pro-cyclical R&D cycle requires low labor growth rate m and low vertical R&D growth rate λ^h. This condition is consistent with the existence of the period 2 cycle.

To summarize the parameter conditions of the local stability of the BGP (17), the existence of the period 2 cycle (18), and pro-cyclical R&D cycle (19), we have the following table.

local stability of the BGP	$e-1 < m\lambda^h$
existence of the period 2 cycle	$m\lambda^h \leq \left(1+\frac{1}{\mu C}-\alpha^{\frac{\alpha}{\alpha-1}}\right)^{\frac{1}{2}}$
pro-cyclical R&D cycle	$m\lambda^h < \left[\mu\left(\frac{1-\alpha}{F}\left(\frac{\alpha F}{a(1-\alpha)}\right)^{\alpha}\right)\beta L_0^{1-\beta}K_0^{\beta-1}\right]^{\frac{1}{\beta}}$

table 2

4 Conclusion

We construct a growth cycle model that contains labor growth, horizontal R&D cycle, and vertical R&D. We verify that the model faces with pro-cyclical R&D cycle in which average growth rate is stable beyond ages when labor growth rate and vertical R&D growth rate are not high. Moreover, R&D promotion policy can stimulate both short-run and long-run growth rate. The main contribution of this article from previous studies, especially from Yano and Kanehara (2016), lies in the long-run implication of R&D promotion policy and (average) upward trend in R&D expenditure. In Yano and Kanehara (2016) they consider labor growth and horizontal R&D cycle, and neglect vertical R&D. On the other hand, our model considers vertical R&D as Dinopoulos and Thompson (1998), Peretto (1998) and Howitt (1999) do. Therefore, our model is a fully-endogenous growth cycle model without scale effect. There are many fully-endogenous Schumpeterian growth models without scale effect, which consider labor growth, and vertical R&D; however, few of them consider the cyclical behavior in R&D and economic growth. On the other hand, there are many growth cycle models, most of which analyze counter-cyclical R&D cycle except some recent studies such as Walde (2005), and Barlevy (2007). Based on Yano and Kanehara (2016), which has good chances to face with pro-cyclical R&D cycle, our model also explains pro-cyclical R&D cycle and long-run implication of R&D promotion policy. In addition to these, while most of all fully-endogenous growth models without scale effect are suffered by the knife-edge critique pointed out by Jones (1999), since horizontal and vertical R&D consume different inputs (capital and labor) in our model, both the BGP and the period 2 cycle are free from such critique.

Though our model assumes fixed saving rate as Solow (1956) does, if we assume labor growth, however, it is natural to endogenize the fertility in the next step, and it requires the optimization of representative household which controls the number of children and consumption. There are many endogenous growth models which consider both endogenous fertility and R&D like Jones (2003). However, only a few of recent works such as Strulik et al. (2013) and Chu et al. (2013) examine both the quantity-quality trade-off in reproduction and R&D simultaneously. Since our model assume that each R&D requires heterogeneous types of inputs; capital and labor (engineers), there is a good chance to consider the the quantity-quality trade-off in reproduction like Galor (2011). Introducing the quantity-quality trade-off in reproduction into our model may contribute such literature because cyclical behavior in R&D sector can affect the quantity-quality trade-off, and might explain overeducation[12] by the endogenous fluctuation in demand for human capital through R&D cycle.

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[12] The literature of overeducation is summarized in McGuinness (2006). However, few studies can explain why overeducation occurs endogenously.

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